Ratios, Rates, and Proportions

Knowledge of solving linear equations is required for this chapter.
See Linear Relationships: Expressions and Equations

Ratios

A **ratio** is a comparison of quantities. Ratios are used to show how much of one quantity there is compared to another quantity.

A bowl contains 7 cherries and 12 blueberries.
The ratio of cherries to blueberries is 7 to 12.

There are 15 students and 2 teachers in a class.
The ratio of students to teachers is 15 to 2.

A ratio compares quantities by division. It is possible to express a ratio in three different ways.

	ratio	odds form	fraction form
general	a to b	$a : b$	$\frac{a}{b}$
cherries to blueberries	7 to 12	7 : 12	$\frac{7}{12}$
students to teachers	15 to 2	15 : 2	$\frac{15}{2}$

Example

A box contains 25 nails, 13 screws, and 8 hooks. What is the ratio of hooks to nails?

Explanation

There are 8 hooks and 25 nails, so the ratio of hooks to nails is **8** to **25**.

Each of the ratios just described compared a part to a part. Ratios can also describe a part to a whole.

A bowl contains 7 cherries and 12 blueberries.

7 + 12 = 19 pieces of fruit.

↓

The ratio of cherries to pieces of fruit is 7 to 19.

There are 15 students and 2 teachers in a class.

15 + 2 = 17 people.

↓

The ratio of teachers to total people is 15 to 17.

Practice 1

Consider the same box that contains 25 nails, 13 screws, and 8 hooks. If these are the only hardware in the box, what is the ratio of screws to the total number of hardware pieces?

Just like fractions, there are an **infinite number of equivalent ratios to any given ratio**. If the ratio is in simplified form, the numbers in the ratio may be different than the actual quantities.

A **ratio of *a* to *b*** means that the first quantity ***a* is multiplied by some number** and the second quantity ***b* is multiplied by that same number**. That number is the scale factor of the ratio.

simplified ratio	actual quantities
$a : b$	an and bn

where n is the scale factor

Example

In a math competition, the ratio of points that Nick earned to the number of points that Henry earned was 4 : 3. Nick and Henry earned a total of 84 points. How many points did Henry earn?

Explanation

The given ratio (4 : 3) means that Nick's points are 4 times some number n and Henry's points are 3 times that same number n. Use the given total number of points (84) to write an equation.

$$\underbrace{4n}_{\text{Nick's points}} + \underbrace{3n}_{\text{Henry's points}} = \underbrace{84}_{\text{total points}}$$

Now solve the equation for n.

$$\begin{aligned} 4n + 3n &= 84 \\ 7n &= 84 \quad \text{Combine } n\text{-terms} \\ n &= 12 \quad \text{Divide by 7 on both sides} \end{aligned}$$

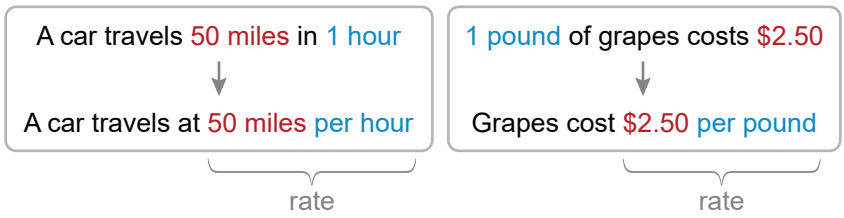
The scale factor n is 12, so multiply 3(12) = 36 to see that Henry earned **36 points**.

Practice 2

The sum of 2 positive integers is 52, and the ratio of the integers is 5 : 8. What is the value of the larger of the two integers?

Rates

A **rate** is a type of ratio used to compare two quantities that have **different units**. Some common types of rates include speed and cost.



Example

Michael swam for 30 minutes and burned 240 calories. How many calories did he burn per minute?

Explanation

The desired value is the rate of calories burned per minute. It is given that Michael burned 240 calories in 30 minutes. Write this as a ratio and simplify.

$$\frac{\text{calories}}{\text{minute}} \rightarrow \frac{240 \text{ calories}}{30 \text{ minutes}} \rightarrow \frac{8 \text{ calories}}{1 \text{ minute}}$$

Michael burned **8 calories per minute**.

Questions may involve multiple rates. Make sure to carefully read the question to determine which rate applies to which amount.

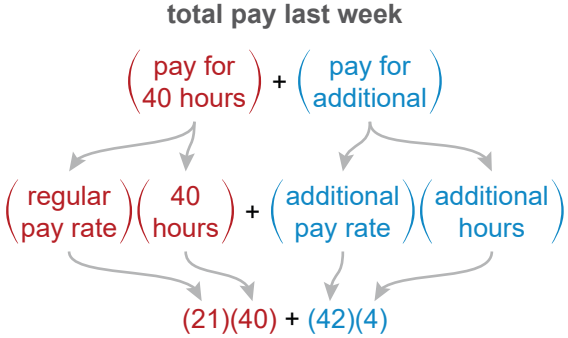
Example

Sharon's regular pay rate is $21 per hour for working up to 40 hours in 1 week. For each additional hour she works in a week, Sharon is paid 2 times her regular hourly pay. Sharon worked 44 hours last week. What was her pay for last week?

Explanation

The amount paid at each rate is the number of hours multiplied by the pay rate for those hours.

Sharon worked 44 hours, so after the first 40 hours she worked 4 additional hours. Her pay rate for the additional hours is 2 times the regular hourly pay ($21), so she is paid $42 per additional hour.



Simplify (21)(40) + (42)(4) to see that Sharon's pay last week was **$1,008**.

Practice 3

A 900-square-foot apartment costs $1,125 per month to rent. What is the monthly cost, per square foot, to rent this apartment?

Proportions

When two ratios are equivalent, they form a proportion. A **proportion** is an equation of the form $\frac{a}{b} = \frac{c}{d}$, where the ratios $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent.

$\frac{1}{2}$ and $\frac{2}{4}$

these ratios are equivalent, so they do form a proportion

$\frac{1}{2} = \frac{2}{4}$

$\frac{1}{3}$ and $\frac{2}{5}$

these ratios are <u>not</u> equivalent, so they do <u>not</u> form a proportion

$\frac{1}{3} \neq \frac{2}{5}$

Example

The ratio of 2 to 3 is equal to the ratio of *x* to 12. What is the value of *x*?

Explanation

It is given that the two ratios are equal, so they form a proportion. Set them equal and solve for *x*.

$$\frac{2}{3} = \frac{x}{12} \quad \text{Set ratios equal}$$

$$(12)\frac{2}{3} = (12)\frac{x}{12} \quad \text{Multiply by 12 on both sides}$$

$$8 = x \quad \text{Simplify}$$

Practice 4

Given that 18 : 15 is equal to 30 : *n*, what is the value of *n* ?

Practice 5

At a specialty spice shop, Cassie spent $8 to buy cinnamon at a price of $2 per ounce. How many ounces of cinnamon did Cassie buy?