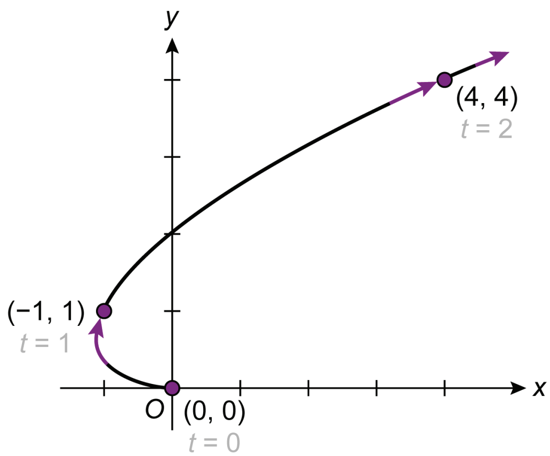
Topic 9.1

Defining and Differentiating Parametric Equations

Parametric Equations and Derivatives

Parametric equations are functions of some variable t that give the x- and y-coordinates on the graph of a curve. Parametric functions are denoted as the **ordered pair $(x(t), y(t))$** of the functions that define its coordinates.

The graph of the curve represented by parametric equations gives all points $(x(t), y(t))$ for any value of t, and it is sometimes given with arrows on the curve to denote the **direction of increasing t**. The graph below shows a parametric curve defined by $(t^3 - 2t, t^2)$.



t	$x(t) = t^3 - 2t$	$y(t) = t^2$
0	0	0
1	−1	1
2	4	4

The derivatives of the coordinate functions $\frac{dx}{dt}$ and $\frac{dy}{dt}$ give the rates of change of each coordinate with respect to the variable t, or the **change in x** and the **change in y** as t changes.

The slope of the curve in the xy-plane is given by $\frac{dy}{dx}$, the derivative of y with respect to x. The function x is a function of t, so use the **chain rule** to relate these three derivatives: $\frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{dy}{dt}$. Divide both sides by $\frac{dx}{dt}$ to get an expression for the **slope of the tangent line** $\frac{dy}{dx}$.

derivative of parametric curve

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

As shown above, a parametric curve may have vertical tangent lines if $\frac{dy}{dt}$ is finite and nonzero and $\frac{dx}{dt} = 0$.

9.1 Check for Understanding

1. **A curve in the *xy*-plane is defined by the parametric equations $x(t) = \sqrt{t}$ and $y(t) = t^2$ for $t > 0$. Which of the following is an expression for $\frac{dy}{dx}$?**

   A. $\sqrt{t}$

   B. $\sqrt{t^3}$

   C. $4\sqrt{t}$

   D. $4\sqrt{t^3}$

2. **For the curve defined by parametric equations $x(t) = 2t^2 - 5t + 1$ and $y(t) = \ln t$ for $t > 0$, which of the following has the greatest value when $t = 1$?**

   A. The instantaneous rate of change of the *x*-coordinate with respect to *t*

   B. The instantaneous rate of change of the *y*-coordinate with respect to *t*

   C. The slope of the line tangent to the curve