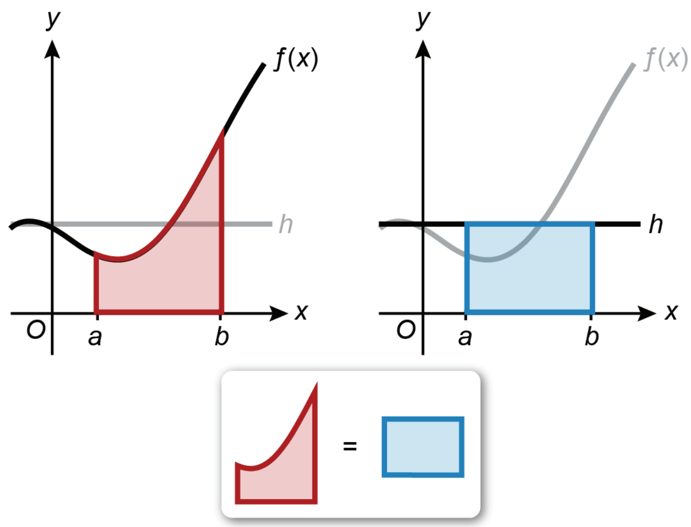
Topic 8.1

Finding the Average Value of a Function on an Interval

Average Value with Definite Integrals

The **average value** of a continuous function *f* over a closed interval [*a*, *b*] is the value that would result if every value on the interval was averaged together. This process is impossible to do directly because an infinite number of function values exist on the interval.

However, consider that the average function value is the **average *y*-value** of the graph of $y = f(x)$, which gives the **average height *h*** of the graph over [*a*, *b*]. The **area under the curve** of *f* equals the **area of a rectangle** with height *h* and base length equal to the interval width.



The area under the graph of *f* is given by the **definite integral** of *f* on [*a*, *b*], and the area under the constant function is a **rectangle** with height *h* and width *b* − *a*, or the width of the interval.

These two areas are equal, so it is possible to solve for the average value *h*.

$$\begin{array}{c} \text{area under graph} \\ \text{of } f \text{ on } [a, b] \end{array} = \begin{array}{c} \text{area of rectangle with} \\ \text{height } h \text{ and width } b - a \end{array} \qquad \text{Set areas equal}$$

$$\int_a^b f(x)\,dx = h(b - a) \qquad \text{Substitute expressions for areas}$$

$$\frac{1}{b - a}\int_a^b f(x)\,dx = h \qquad \text{Divide both sides by } b - a$$

Therefore, the average value of *f* on [*a*, *b*] is given by the definite integral of *f* on [*a*, *b*] divided by *b* − *a*.

average value

$$\frac{1}{b - a}\int_a^b f(x)\,dx$$

8.1 Check for Understanding

$$f(x) = \begin{cases} 2x - 1, & \text{for } x < 2 \\ \sin(\pi x), & \text{for } x \geq 2 \end{cases}$$

1. **Which of the following is the average value of the function *f* on the interval [1, 3] ?**

   A. 0

   B. 1.5

   C. 3

   D. The average value of *f* on [1, 3] cannot be found.

2. **What is the average value of the function $f(x) = x^2 + 2x - 4$ on the interval [0, 3] ?**

   A. 2

   B. 5

   C. 6

   D. The average value of *f* on [0, 3] cannot be found.