Topic 6.1

Exploring Accumulations of Change

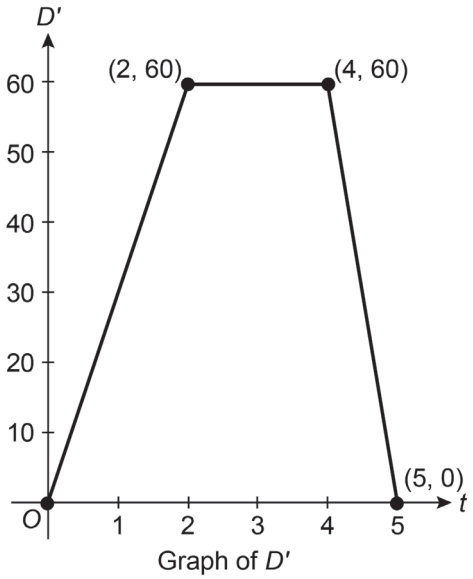
Accumulation and Area

The **accumulation of change** (or **net change**) over an interval in a quantity changing over time is given by the **area enclosed by the graph** of the rate of change.

Example

A car's position along a straight road over a five-hour trip is given by the function $D(t)$, where D is the number of miles away from the starting point t hours into the trip.

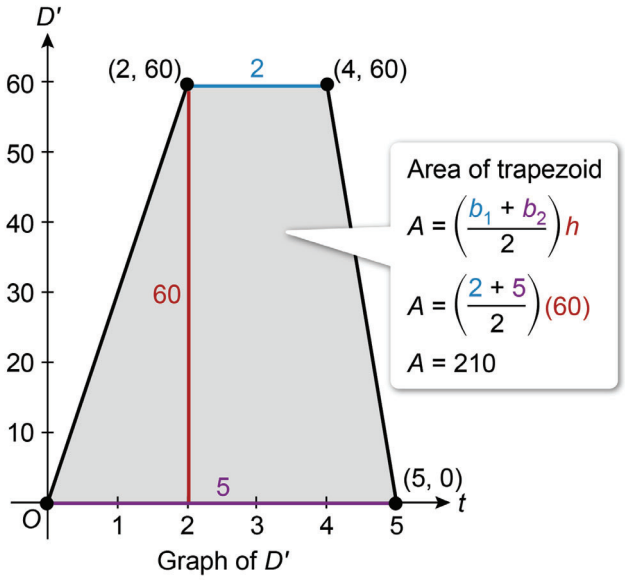
The graph below gives the car's velocity $D'(t)$, in miles per hour, over the five-hour interval [0, 5].



Graph of D'

When a quantity changes at a constant rate, it is possible to find the net change with the distance formula. However, when the rate is not constant, find the **accumulation** by calculating the **area under the curve**.

The **total number of miles** that the car travels (or **net change** in the car's position) over the five-hour trip is given by the **total area** under the graph of D' on [0, 5]. The area under the graph of D' is a trapezoid, so apply the formula for the area of a trapezoid to evaluate.



Graph of D'

Over the five-hour trip, the car traveled **210 miles**.

The **units of an accumulation** of a quantity are the units of the quantity, which are given by the product of the units of the rate of change and the units of the independent variable.

Notice that in the example above, the units of distance are miles, the units of speed are miles per hour, and the units of time are hours.

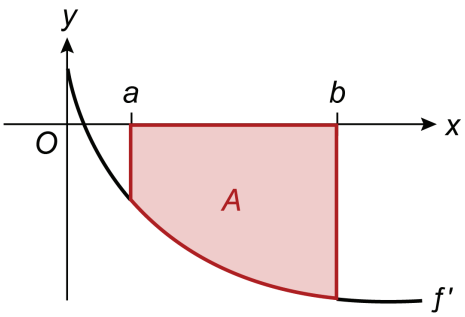
units of accumulation

units of rate · units of time = $\frac{\text{miles}}{\text{hour}} \cdot \text{hours} = \text{miles}$

Negative Accumulation

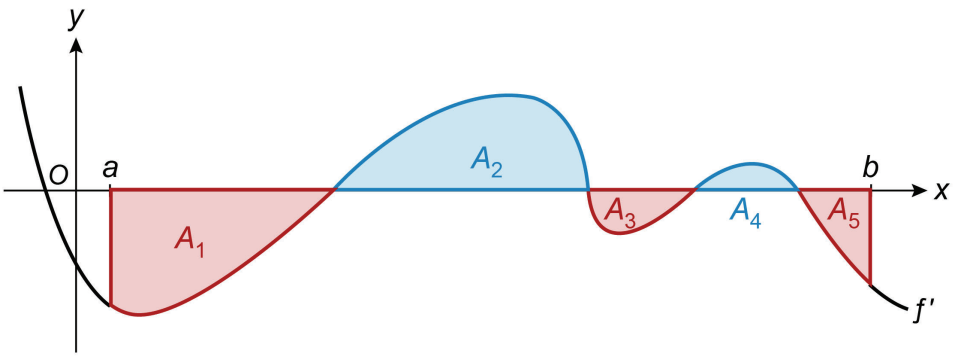
If the rate of change in a quantity is **negative** (derivative graph is below the *x*-axis) over an interval, then the **accumulation is negative** (the quantity is decreasing) over that interval.

Area is always positive, so the accumulation of a quantity with a negative rate of change is given by the negative of the area under the curve.



accumulation of f on $[a, b]$ $= -A$

Therefore, the total accumulation of a quantity over an interval is given by the total area bounded by the graph of the rate of change above the x-axis minus the total area below the x-axis.



$$\text{accumulation} = \left(\begin{array}{c}\text{total area}\\ \text{above } x\text{-axis}\end{array}\right) - \left(\begin{array}{c}\text{total area}\\ \text{below } x\text{-axis}\end{array}\right)$$
$$= (A_2 + A_4) - (A_1 + A_3 + A_5)$$

6.1 Check for Understanding

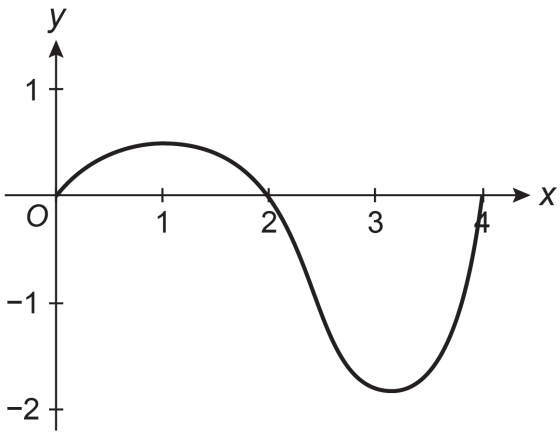
1. **The height, in feet, of an object tossed into the air is given by the function $h(t) = -3t^2 + 12t$ on the interval [0, 4], where time t is measured in seconds since the object was tossed. Which of the following is the net change in the object's height over the first 2 seconds of its flight?**

   A. 0

   B. 6

   C. 12

   D. 16



Graph of Q'

2. **The graph of the rate of change in a quantity Q over the interval [0, 4] is shown above. Which of the following are positive values?**

   I. The accumulation of Q over [0, 2]

   II. The accumulation of Q over [0, 4]

   III. The accumulation of Q over [2, 4]

   A. I only

   B. III only

   C. I and II only

   D. II and III only