

Topic 5.1

Using the Mean Value Theorem

The MVT

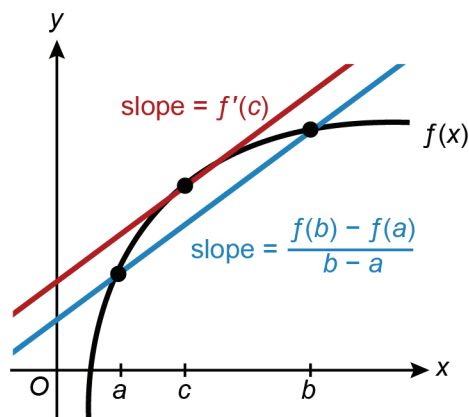
The **Mean Value Theorem (MVT)** states that if a function f is continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b) , then there exists a value c in (a, b) such that the instantaneous rate of change $f'(c)$ equals the average rate of change of f on $[a, b]$.

Mean Value Theorem

Conditions	Conclusion
f is continuous on $[a, b]$	for some c in (a, b) :
f is differentiable on (a, b)	$f'(c) = \frac{f(b) - f(a)}{b - a}$ IROC at $x = c$ AROC on $[a, b]$

The IROC $f'(c)$ corresponds to the **slope of the line tangent** to f at $x = c$, and the AROC $\frac{f(b) - f(a)}{b - a}$ corresponds to the **slope of the secant line** between $(a, f(a))$ and $(b, f(b))$.

If f meets the conditions of the MVT, then at least one line tangent to f on the interval (a, b) is **parallel** to the secant line between the endpoints of the interval. The following graph shows a function f on an interval $[a, b]$ that meets the conditions of the MVT.



A special case of the MVT called **Rolle's theorem** helps find intervals on which the derivative of a function is 0. Notice that the AROC of a function f over an interval $[a, b]$ is 0 when $f(a) = f(b)$.

If a function f meets the conditions of the MVT on an interval $[a, b]$ and $f(a) = f(b)$, then there exists a value c in (a, b) for which $f'(c) = 0$.

Rolle's Theorem

Conditions	Conclusion
f is continuous on $[a, b]$	for some c in (a, b) : $f'(c) = 0$
f is differentiable on (a, b)	
$f(a) = f(b)$	

If a function f is continuous and differentiable on an interval, then the derivative of f must equal 0 and the **tangent line is horizontal** at least once between any two points on the interval whose function values are equal.

5.1 Check for Understanding

1. The Mean Value Theorem guarantees that there must be a value c in $(-1, 2)$ such that the slope of the line tangent to the function $f(x) = \cos(\pi x)$ equals $\frac{2}{3}$.

True

False

$$f(x) = \begin{cases} \sqrt{-x} & \text{for } x < 0 \\ \sqrt{x} & \text{for } x \geq 0 \end{cases}$$

2. Let f be the piecewise function defined above. Which of the following conditions of Rolle's theorem is NOT satisfied on the interval $[-1, 1]$?

I. f is continuous on $[-1, 1]$

II. f is differentiable on $[-1, 1]$

III. $f(-1) = f(1)$

A. I only

B. II only

C. III only

D. I and II only