Topic 5.1

Using the Mean Value Theorem

The MVT

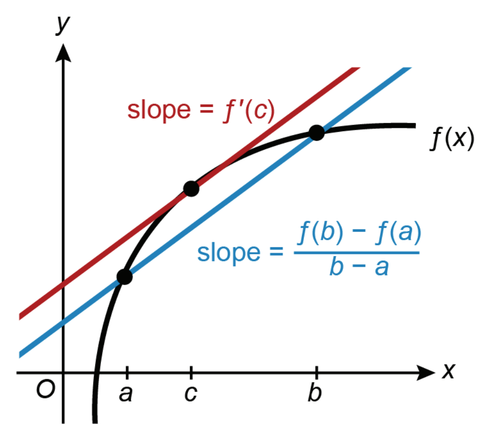
The **Mean Value Theorem (MVT)** states that if a function *f* is continuous on a closed interval [*a*, *b*] and differentiable on the open interval (*a*, *b*), then there exists a value *c* in (*a*, *b*) such that the instantaneous rate of change $f'(c)$ equals the average rate of change of *f* on [*a*, *b*].

Mean Value Theorem

Conditions	Conclusion
f is continuous on [*a*, *b*]	for some *c* in (*a*, *b*): $f'(c) = \frac{f(b) - f(a)}{b - a}$
f is differentiable on (*a*, *b*)	IROC at $x = c$; AROC on [*a*, *b*]

The IROC $f'(c)$ corresponds to the **slope of the line tangent** to *f* at $x = c$, and the AROC $\frac{f(b)-f(a)}{b-a}$ corresponds to the **slope of the secant line** between (*a*, *f*(*a*)) and (*b*, *f*(*b*)).

If *f* meets the conditions of the MVT, then at least one line tangent to *f* on the interval (*a*, *b*) is **parallel** to the secant line between the endpoints of the interval. The following graph shows a function *f* on an interval [*a*, *b*] that meets the conditions of the MVT.



A special case of the MVT called **Rolle's theorem** helps find intervals on which the derivative of a function is 0. Notice that the AROC of a function *f* over an interval [*a*, *b*] is 0 when $f(a) = f(b)$.

If a function *f* meets the conditions of the MVT on an interval $[a, b]$ and $f(a) = f(b)$, then there exists a value *c* in (a, b) for which $f'(c) = 0$.

Rolle's Theorem

Conditions	Conclusion
f is continuous on $[a, b]$	for some c in (a, b):
f is differentiable on (a, b)	$f'(c) = 0$
$f(a) = f(b)$	

If a function *f* is continuous and differentiable on an interval, then the derivative of *f* must equal 0 and the **tangent line is horizontal** at least once between any two points on the interval whose function values are equal.

5.1 Check for Understanding

1. **The Mean Value Theorem guarantees that there must be a value *c* in (−1, 2) such that the slope of the line tangent to the function $f(x) = \cos(\pi x)$ equals $\frac{2}{3}$.**

   True

   False

$$f(x) = \begin{cases} \sqrt{-x} & \text{for } x < 0 \\ \sqrt{x} & \text{for } x \geq 0 \end{cases}$$

2. **Let *f* be the piecewise function defined above. Which of the following conditions of Rolle's theorem is NOT satisfied on the interval [−1, 1] ?**

   I. *f* is continuous on [−1, 1]

   II. *f* is differentiable on [−1, 1]

   III. *f*(−1) = *f*(1)

   A. I only

   B. II only

   C. III only

   D. I and II only