Topic 4.1

Interpreting the Meaning of the Derivative in Context

Derivatives in Context

The derivative $\frac{dy}{dx}$ represents the **instantaneous rate of change (IROC)** of a variable *y* with respect to another variable *x*, which is the rate at which *y* changes at an instant or a particular value of *x*.

instantaneous rate of change

$$f'(x) = \frac{dy}{dx} = \frac{\text{change in } y}{\text{change in } x}$$

The derivative $\frac{dy}{dx}$ corresponds to the slope of the line tangent to *y*, which gives **the change in *y* divided by the change in *x***. Therefore, the units of $\frac{dy}{dx}$ are $\frac{\text{units for } y}{\text{units for } x}$.

Example 1

The function *W*(*t*) represents the weight of a newborn bear cub in ounces *t* days after it is born.

The rate at which the cub is growing at time *t* is given by $\frac{dW}{dt}$, which represents the change in weight *W* over the change in time *t*. Therefore, the units of $\frac{dW}{dt}$ are the units of *W* over the units of *t*, or $\frac{\text{ounces}}{\text{day}}$.

$$\frac{dW}{dt} \rightarrow \frac{\text{units for } W}{\text{units for } t} = \frac{\text{ounces}}{\text{day}}$$

Example 2

The function *S*(*t*) represents the speed at which a pulley rotates in radians per second *t* seconds after it begins rotating.

As time passes (*t* increases), the pulley rotates faster (*S* increases). The rate at which the speed of the pulley decreases is given by $\frac{dS}{dt}$. The units of $\frac{dS}{dt}$ are the units of *S* over the units of *t*:

$$\frac{dS}{dt} \rightarrow \frac{\text{units for } S}{\text{units for } t} = \frac{\text{radians/second}}{\text{second}}$$

The resulting units are $\frac{\text{radians per second}}{\text{second}}$, which simplifies to $\frac{\text{radians}}{(\text{second})^2}$.

4.1 Check for Understanding

1. **A faucet fills a bathtub with water. The amount of water in the bathtub at time *t* seconds after turning on the faucet is modeled by a differentiable function *w*, measured in milliliters. Which of the following is the best interpretation of *w′*(10) ?**

   A. The amount of water, in milliliters, that has entered the bathtub during the first 10 seconds after the faucet is turned on.

   B. The rate at which water fills the bathtub, in milliliters per second, 10 seconds after the faucet is turned on.

   C. The amount of change, in milliliters per second, in the rate at which water enters the bathtub 10 seconds after the faucet is turned on.

   D. The rate of change in the rate at which water enters the bathtub, in milliliters per second per second, 10 seconds after the faucet is turned on.

2. **The pressure, in psi, in a water heater *t* hours after it is turned on is modeled by the function $P(t) = t^2 + \sqrt{t}$ for $0 \le t \le 8$. Which of the following is the rate at which the pressure is changing, in psi per hour, at time $t = 4$?**

   A. $\frac{15}{4}$

   B. $\frac{33}{4}$

   C. 9

   D. 18