

Topic 4.1

Interpreting the Meaning of the Derivative in Context

Derivatives in Context

The derivative $\frac{dy}{dx}$ represents the **instantaneous rate of change (IROC)** of a variable y with respect to another variable x , which is the rate at which y changes at an instant or a particular value of x .

instantaneous rate of change

$$f'(x) = \frac{dy}{dx} = \frac{\text{change in } y}{\text{change in } x}$$

The derivative $\frac{dy}{dx}$ corresponds to the slope of the line tangent to y , which gives **the change in y divided by the change in x** . Therefore, the units of $\frac{dy}{dx}$ are $\frac{\text{units for } y}{\text{units for } x}$.

Example 1

The function $W(t)$ represents the weight of a newborn bear cub in ounces t days after it is born.

The rate at which the cub is growing at time t is given by $\frac{dW}{dt}$, which represents the change in weight W over the change in time t . Therefore, the units of $\frac{dW}{dt}$ are the units of W over the units of t , or $\frac{\text{ounces}}{\text{day}}$.

$$\frac{dW}{dt} \rightarrow \frac{\text{units for } W}{\text{units for } t} = \frac{\text{ounces}}{\text{day}}$$

Example 2

The function $S(t)$ represents the speed at which a pulley rotates in radians per second t seconds after it begins rotating.

As time passes (t increases), the pulley rotates faster (S increases). The rate at which the speed of the pulley decreases is given by $\frac{dS}{dt}$. The units of $\frac{dS}{dt}$ are the units of S over the units of t :

$$\frac{dS}{dt} \rightarrow \frac{\text{units for } S}{\text{units for } t} = \frac{\text{radians/second}}{\text{second}}$$

The resulting units are $\frac{\text{radians per second}}{\text{second}}$, which simplifies to $\frac{\text{radians}}{(\text{second})^2}$.

4.1 Check for Understanding

1. **A faucet fills a bathtub with water. The amount of water in the bathtub at time t seconds after turning on the faucet is modeled by a differentiable function w , measured in milliliters. Which of the following is the best interpretation of $w'(10)$?**
 - A. The amount of water, in milliliters, that has entered the bathtub during the first 10 seconds after the faucet is turned on.
 - B. The rate at which water fills the bathtub, in milliliters per second, 10 seconds after the faucet is turned on.
 - C. The amount of change, in milliliters per second, in the rate at which water enters the bathtub 10 seconds after the faucet is turned on.
 - D. The rate of change in the rate at which water enters the bathtub, in milliliters per second per second, 10 seconds after the faucet is turned on.
2. **The pressure, in psi, in a water heater t hours after it is turned on is modeled by the function $P(t) = t^2 + \sqrt{t}$ for $0 \leq t \leq 8$. Which of the following is the rate at which the pressure is changing, in psi per hour, at time $t = 4$?**
 - A. $\frac{15}{4}$
 - B. $\frac{33}{4}$
 - C. 9
 - D. 18