Topic 2.1

Systems and Center of Mass

Learning Objectives

- Describe the properties and interactions of a system.
- Describe the location of a system's center of mass with respect to the system's constituent parts.

Topic Questions

- What is a system and how is it defined?
- What does the center of mass represent?
- How can the position of the center of mass of a system be determined?

2.1.01 Properties of Systems

[2.1.A.1 2.1.A.2 2.1.A.3 2.1.A.4 2.1.A.5 2.1.A.6]

An **object** is an isolated collection of particles treated as having no internal structure. In other words, the interactions between the particles that make up an object can be ignored for the purpose of analyzing the overall motion of the object.

Furthermore, a **system** is a collection of objects that interact in ways that influence the other objects in the system. For example, a baseball is made up of many protons, neutrons, and electrons that comprise a system (Figure 2.1). However, when a force is exerted on the baseball (hit by a bat or thrown by a player), the resulting motion of the baseball is not affected by the interactions of the particles that make up the ball.

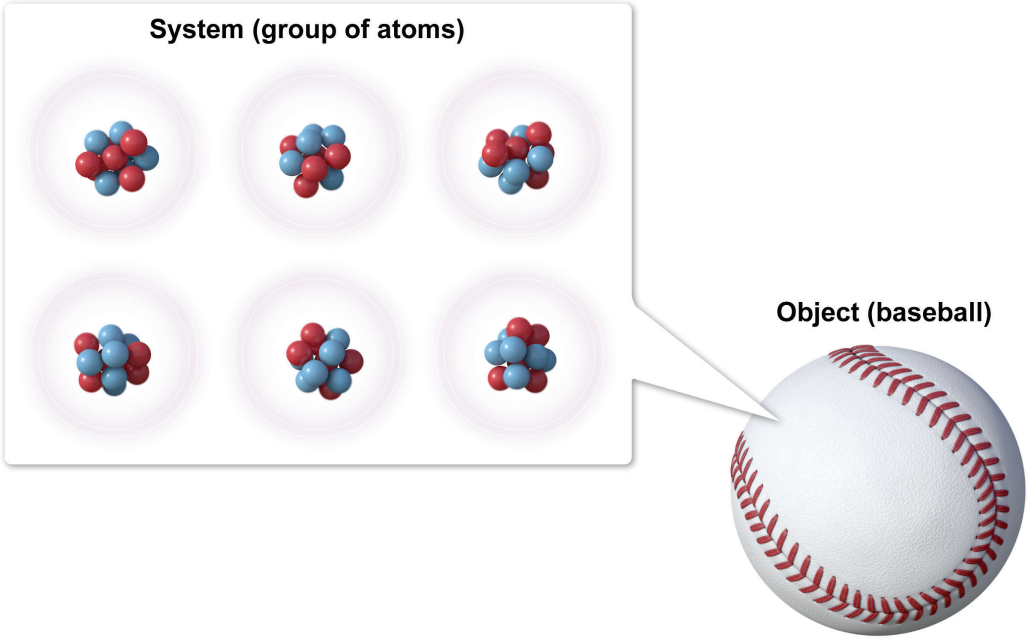

Figure 2.1 The baseball is an object because internal microscopic interactions do not affect overall system motion.

Therefore, a baseball is considered an object with no internal structure because the motion of individual protons, neutrons, and electrons in the baseball is not relevant to the motion of the ball.

Furthermore, consider a system of four balls such as those a juggler tosses into the air, as shown in Figure 2.2.

Figure 2.2 Four balls can be considered a system of objects with a center of mass CM.

The four balls can be defined as objects within a system under the influence of external forces such as gravity and air resistance. Consequently, the four-ball system can also be characterized by a **center of mass (CM)** position, which can be used to represent the system's external interactions.

2.1.02 Determining Center of Mass Location

[2.1.B.1 2.1.B.2 2.1.B.3]

The mass of an entire system of objects can be characterized by a point in space known as the **center of mass (CM)**. The CM position x_{CM} represents the average of each object's displacement x_i from a reference point weighted by their mass m_i:

$$x_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

The CM position represents the entire system as one combined mass concentrated at a single point. This position is influenced more by the heavier objects included in the system.

Consider a scenario with two balls, a baseball of mass m_1 and a basketball of mass $m_2 = 2m_1$, as shown in Figure 2.3. If m_1 is located at position $x = 0$ and m_2 is located at $x = x_2$, the distance of each ball to the CM can be determined:

$$x_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{m_1(0) + (2m_1)x_2}{m_1 + (2m_1)}$$

$$x_{CM} = \frac{2\cancel{m_1}x_2}{3\cancel{m_1}} = \frac{2}{3}x_2$$

Hence, the distance d_1 of m_1 to the center of mass is equal to $\frac{2}{3}x_2$, and the distance d_2 of m_2 to the center of mass must be the remaining $\frac{1}{3}x_2$, which is half the distance away that m_1 is:

$$d_2 = \frac{1}{2}d_1$$

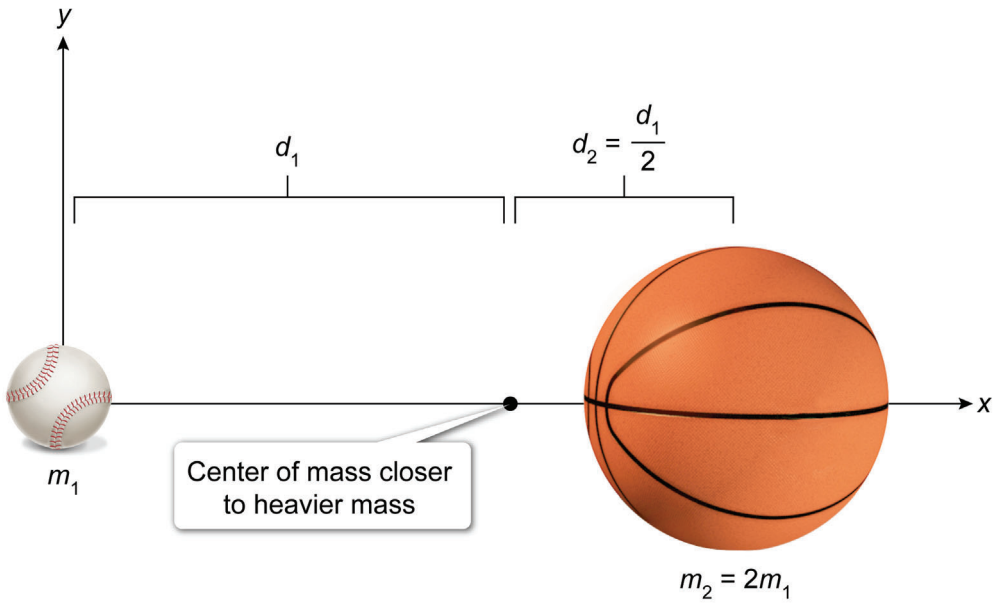

Figure 2.3 The CM of a system is closer to objects of greater mass.

As can be seen in the example, the CM position is closer to the heavier object by a factor equal to the ratio of the masses of each object. Moreover, the CM position for an extended object also abides by these relationships.

Consider the human body, as shown in Figure 2.4, where the CM position is closer to the areas of the body that contain more mass.

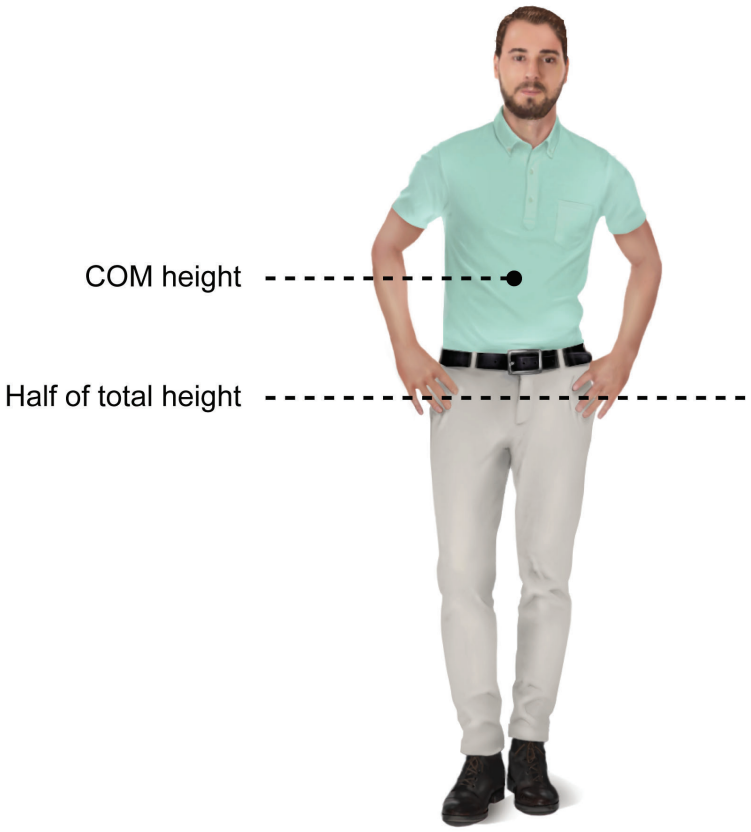

Figure 2.4 The CM position for a person is typically located above the vertical midpoint.

Furthermore, the CM position reduces the calculations that would consider every particle in the system or extended object to that of a single object. Therefore, when a force acts on a system, it can be considered as acting only on the system's CM.

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Check for Understanding Quiz

1. A student analyzing different physical scenarios is trying to determine whether to treat the subjects of the scenarios as individual objects or as systems. Which of the following scenarios is best analyzed by treating the subject as a single object without considering its internal structure?

   A. Investigating the heat distribution within a metal rod when one end is heated.

   B. Examining the motion of a soccer ball after it has been kicked.

   C. Studying the ecological interactions within a pond ecosystem.

   D. Analyzing the electrical circuitry inside a computer.

2. Two objects lie along an x-axis. The first object has a mass of 5 kg and is located at $x = 2$ m. The second object has a mass of 10 kg and is located at $x = 8$ m. What is the position of the center of mass on the x-axis?

   A. 2 m

   B. 4 m

   C. 6 m

   D. 8 m

Note: Answers to this quiz are in the back of the book (appendix).