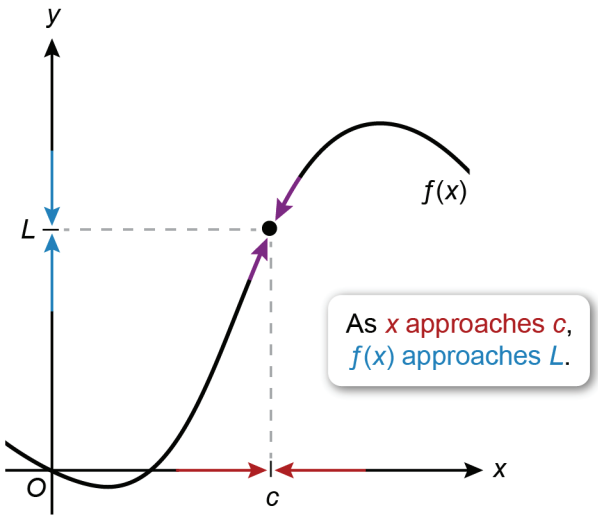
Topic 1.1

Introducing Calculus: Can Change Occur at an Instant?

Limits

Calculus depends on the concept of a **limit**, which is the process of getting arbitrarily close to a point without reaching that point.

A limit examines the behavior of a function as the dependent variable x gets very close to a value c. If the function approaches a particular value L as x approaches c, then the limit equals L.



An important concept in calculus involves using a limit to examine the behavior of the **average rate of change** of a function over an interval as the interval gets smaller and smaller.

Average Rate of Change

The average rate of change (AROC) of a function over an interval is the average amount that the function changes over the length of the interval. It is given by the total change in the function divided by the total change in the interval.

For a function $f(x)$, the AROC over $[a, b]$ is given by $\frac{\text{change in } f}{\text{change in } x} = \frac{f(b)-f(a)}{b-a}$, which also gives the **slope of the secant line** between the points $(a, f(a))$ and $(b, f(b))$.

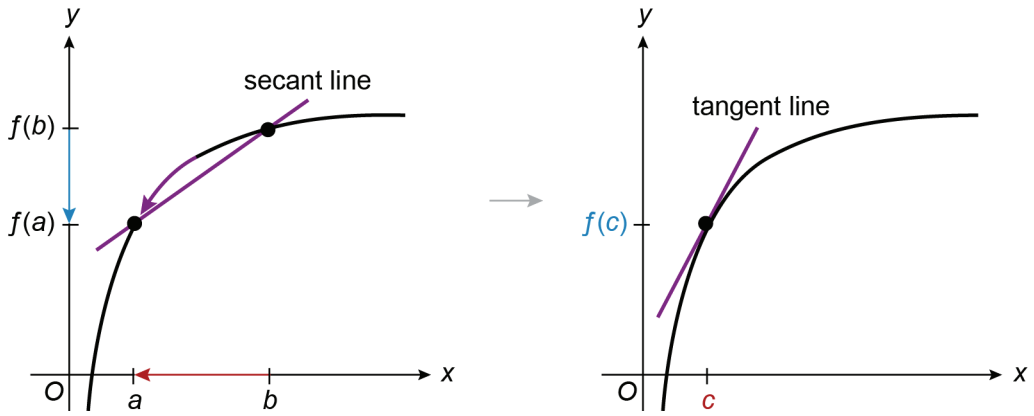
average rate of change over $[a, b]$

$$\frac{f(b) - f(a)}{b - a}$$

Instantaneous Rate of Change

Consider a car that accelerates from 0 miles per hour to 30 miles per hour over an interval of time. The AROC in the car's position gives the **average speed** of the car during the interval.

In contrast, the instantaneous rate of change (IROC) gives the **exact speed** of the car at a particular instant of time within the interval. The IROC at a particular instant c corresponds to the **limit of the AROC** as the endpoints of the interval $[a, b]$ each approach c.



1.1 Check for Understanding

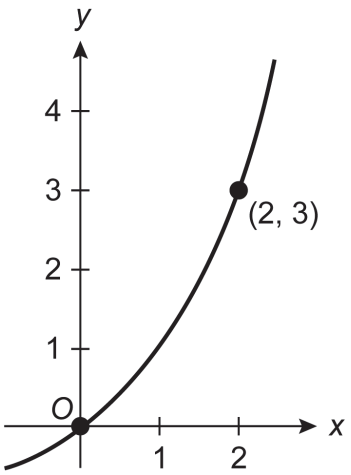
1. **What is the average rate of change in the function $f(x) = x^2 - 2x + 4$ over the interval [1, 3] ?**

   A. 2

   B. 3

   C. 4

   D. 7



Graph of f

2. **The graph of a function f is shown above. What is the average rate of change in f over the interval [0, 2] ?**

   A. $\frac{2}{3}$

   B. $\frac{3}{2}$

   C. 2

   D. 3