Calculus AB Formulas

Section 1: Limits

Properties of limits as $x \to c$

If L, M, c, and k are real numbers and $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$, then the following properties are true:

Limit of a constant	$\lim_{x\to c} k = k$
Limit of x	$\lim_{x\to c} x = c$
Sum rule	$\lim_{x\to c} (f(x) + g(x)) = L + M$
Difference rule	$\lim_{x\to c} (f(x) - g(x)) = L - M$
Product rule	$\lim_{x\to c} (f(x) \cdot g(x)) = L \cdot M$
Constant multiple rule	$\lim_{x\to c} (k(f(x)) = k \cdot L$
Quotient rule	$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{L}{M}$, $M \neq 0$
Power rule	$\lim_{x\to c} (f(x))^{r/s} = L^{r/s}$, if r and s are integers, and $s \neq 0$
Limit of a composite function	$\lim_{x\to c} f(g(x)) = f\left(\lim_{x\to c} g(x)\right)$, if f is a continuous function

Properties of limits as $x \to \pm\infty$

If L, M, c, and k are real numbers and $\lim_{x\to\pm\infty} f(x) = L$ and $\lim_{x\to\pm\infty} g(x) = M$, then the following properties are true:

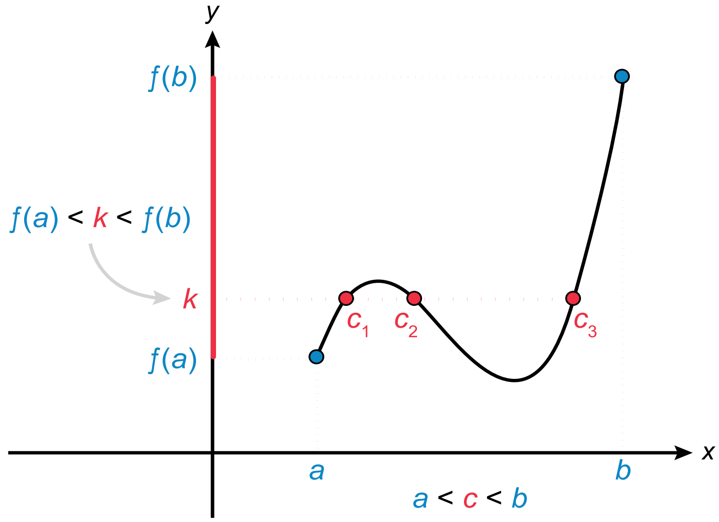
Constant rule	$\lim_{x\to\pm\infty} c = c$
Sum rule	$\lim_{x\to\pm\infty} (f(x) + g(x)) = L + M$
Difference rule	$\lim_{x\to\pm\infty} (f(x) - g(x)) = L - M$
Product rule	$\lim_{x\to\pm\infty} (f(x) \cdot g(x)) = L \cdot M$
Constant multiple rule	$\lim_{x\to\pm\infty} (k(f(x)) = k \cdot L$
Quotient rule	$\lim_{x\to\pm\infty} \frac{f(x)}{g(x)} = \frac{L}{M}$, $M \neq 0$
Power rule	$\lim_{x\to\pm\infty} (f(x))^{r/s} = L^{r/s}$, if r and s are integers and $s \neq 0$
Limit of $\frac{c}{x^r}$	$\lim_{x\to\pm\infty} \frac{c}{x^r} = 0$

squeeze theorem

Conditions	Conclusion
$g(x) \leq f(x) \leq h(x)$ for $x \neq c$	$\lim_{x\to c} f(x) = L$
$\lim_{x\to c} g(x) = L$ and $\lim_{x\to c} h(x) = L$	

Intermediate Value Theorem (IVT)

If a function f is continuous on the interval $[a, b]$ and k is a number between $f(a)$ and $f(b)$, then there is at least one x-value c between a and b such that $f(c) = k$.

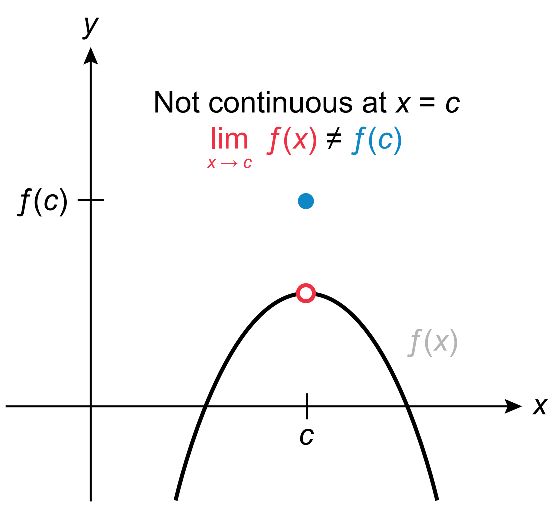


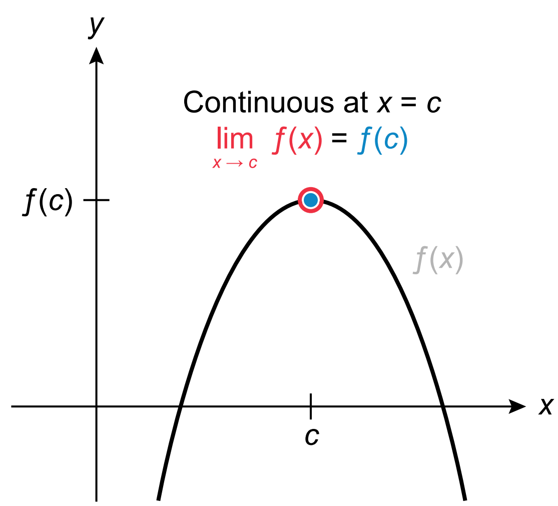
Any continuous function connecting $(a, f(a))$ and $(b, f(b))$ must pass through every y-value between $f(a)$ and $f(b)$ at least once.

A function $f(x)$ is **continuous at $x = c$** if *all* of the following conditions are met:

- $f(c)$ is defined
- $\lim_{x \to c} f(x)$ exists
- $\lim_{x \to c} f(x) = f(c)$

The graph of a continuous function has no "gaps."





Section 2: Derivatives

average rate of change over [a, b]

$$\frac{f(b)-f(a)}{b-a}$$

definition of derivative of f at $x = a$

$$f'(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

definition of derivative of f at $x = a$

$$f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$$

f is differentiable at $x = c$

$\lim_{x \to c} \frac{f(x)-f(c)}{x-c}$ exists and is equal to $f'(c)$

difference quotient

Differentiation rules

Constant	$\frac{d}{dx}[c] = 0$
Power	$\frac{d}{dx}[x^n] = nx^{n-1}$
Natural exponential	$\frac{d}{dx}[e^x] = e^x$
Exponential	$\frac{d}{dx}[a^x] = (\ln a)a^x$
Natural log	$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$
Constant multiple	$\frac{d}{dx}[cf(x)] = cf'(x)$
Sum and difference	$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Derivatives of trigonometric functions

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

$$\frac{d}{dx}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\csc x] = -\csc(x)\cot(x)$$

$$\frac{d}{dx}[\sec x] = \sec(x)\tan(x)$$

product rule

$$\frac{d}{dx}[uv] = uv' + vu'$$

quotient rule

$$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$$

chain rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

derivative of inverse

$$(f^{-1})'(a) = \frac{1}{f'(b)}$$

position	$x(t)$	
velocity	$v(t) = x'(t)$	differentiate
acceleration	$a(t) = x''(t)$	differentiate

Use **L'Hospital's Rule** to find the limit of the ratio of two differentiable functions $\frac{f(x)}{g(x)}$ as x approaches c. If direct substitution produces one of the indeterminate forms $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then differentiate the numerator f and denominator g independently.

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

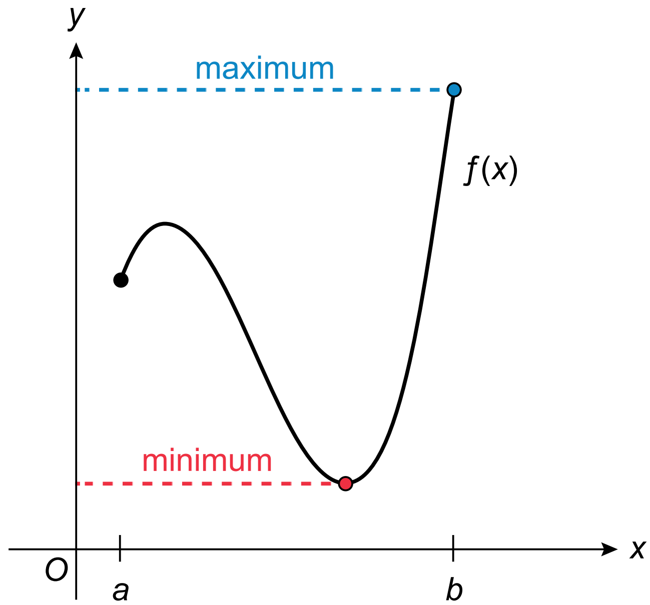
L'Hospital's Rule also applies to limits as $x \to \infty$ or $x \to -\infty$.

Mean Value Theorem

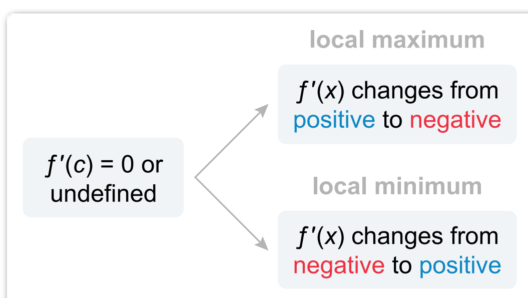
Conditions	Conclusion
f is continuous on $[a, b]$	For some c in (a, b): $f'(c) = \frac{f(b) - f(a)}{b - a}$
f is differentiable on (a, b)	$f'(c)$: instantaneous rate of change at $x = c$; $\frac{f(b) - f(a)}{b - a}$: average rate of change on $[a, b]$

Extreme Value Theorem (EVT)

If a function f is continuous on the closed interval $[a,b]$, then f is guaranteed to attain an absolute minimum and absolute maximum value on $[a,b]$.



local extrema at $x = c$



second derivative test for critical point at $x = c$

If $f''(c) < 0$	If $f''(c) = 0$	If $f''(c) > 0$
$f(c)$ is a relative maximum	test is inconclusive	$f(c)$ is a relative minimum

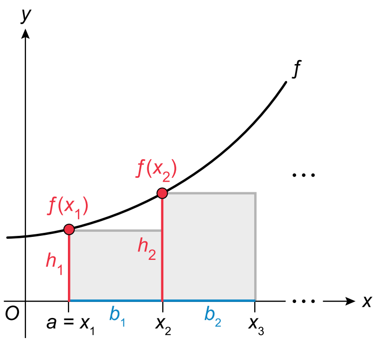
Section 3: Integrals and Differential Equations

A **left Riemann sum** approximates the value of a definite integral $\int_a^b f(x)\,dx$. The interval [a, b] is divided into subintervals, and the area bounded by the graph of f and the x-axis on each subinterval is estimated with a rectangle.

The base length b_n of each rectangle is the distance between the endpoints of the subinterval, and the height h_n is the function value at the left endpoint.

$$\int_a^b f(x)\,dx \approx b_1h_1 + b_2h_2 + \cdots$$

(b_1h_1: area 1st rectangle; b_2h_2: area 2nd rectangle)

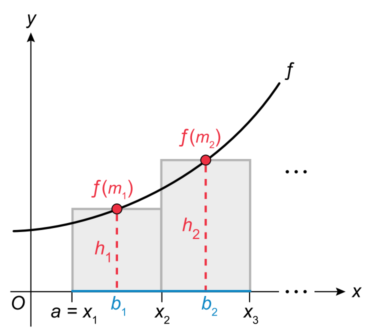


A **midpoint Riemann sum** approximates the value of a definite integral $\int_a^b f(x)\,dx$. The interval [a, b] is divided into subintervals, and the area bounded by the graph of f and the x-axis on each subinterval is estimated with a rectangle.

The base length b_n of each rectangle is the distance between the endpoints of the subinterval, and the height h_n is the function value at the midpoint of the subinterval m_n.

$$\int_a^b f(x)\,dx \approx b_1h_1 + b_2h_2 + \cdots$$

(b_1h_1: area 1st rectangle; b_2h_2: area 2nd rectangle)

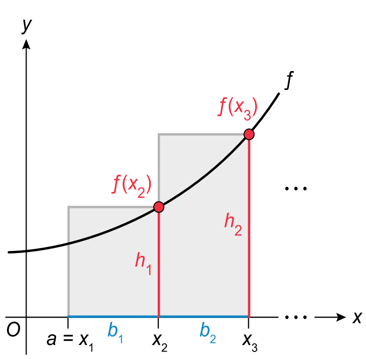


A **right Riemann sum** approximates the value of a definite integral $\int_a^b f(x)\,dx$. The interval [a, b] is divided into subintervals, and the area bounded by the graph of f and the x-axis on each subinterval is estimated with a rectangle.

The base length b_n of each rectangle is the distance between the endpoints of the subinterval, and the height h_n is the function value at the right endpoint.

$$\int_a^b f(x)\,dx \approx b_1h_1 + b_2h_2 + \cdots$$

(b_1h_1: area 1st rectangle; b_2h_2: area 2nd rectangle)

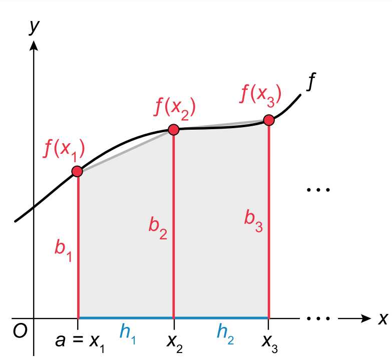


A **trapezoidal sum** approximates the value of a definite integral $\int_a^b f(x)\,dx$. The interval [a, b] is divided into subintervals, and the area bounded by the graph of f and the x-axis on each subinterval is estimated with a trapezoid.

The height h_n of each trapezoid is the distance between the endpoints of the subinterval, and the bases b_n and b_{n+1} are the function values at the endpoints.

$$\int_a^b f(x)\,dx \approx \frac{1}{2}h_1(b_1 + b_2) + \frac{1}{2}h_2(b_2 + b_3) + \cdots$$

($\frac{1}{2}h_1(b_1 + b_2)$: area 1st trapezoid; $\frac{1}{2}h_2(b_2 + b_3)$: area 2nd trapezoid)



Fundamental Theorem of Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

limit of right Riemann sum

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(a + \Delta x i)\Delta x = \int_a^b f(x)\,dx$$

$a(t)$ acceleration

integrate ↓

$v(t)$ velocity

integrate ↓

$s(t)$ position

Second FTC

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

Fundamental Theorem of Calculus (alternate form)

$$f(b) = f(a) + \int_a^b f'(t)\,dt$$

- $f(b)$ → final quantity
- $f(a)$ → initial quantity
- $\int_a^b f'(t)\,dt$ → net change

Integration rules

Constant	$\int c\,dx = cx + C$
Power	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$
Constant multiple	$\int cf(x)\,dx = c\int f(x)\,dx$
Sum and difference	$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
Natural exponential	$\int e^x dx = e^x + C$
Natural log	$\int \frac{1}{x}\,dx = \ln\lvert x\rvert + C$

The following are properties of definite integrals, where functions *f* and *g* are continuous on the closed interval [*a*, *b*] and *a*, *b*, and *k* are constants.

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

$$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

Integrals of trigonometric functions

$$\int \sin u\,du = -\cos u + C$$

$$\int \cos u\,du = \sin u + C$$

$$\int \sec^2 u\,du = \tan u + C$$

$$\int \csc^2 u\,du = -\cot u + C$$

$$\int (\sec u \tan u)\,du = \sec u + C$$

$$\int (\csc u \cot u)\,du = -\csc u + C$$

$$\int \tan u\,du = -\ln|\cos u| + C$$

$$\int \cot u\,du = \ln|\sin u| + C$$

$$\int \sec u\,du = \ln|\sec u + \tan u| + C$$

$$\int \csc u\,du = -\ln|\csc u + \cot u| + C$$

Differential equation

constant of proportionality

$$\frac{dy}{dt} = k \cdot y$$

rate of change of y

↔

General solution

constant of proportionality

$$y = C \cdot e^{k \cdot t}$$

t → time

C → initial value of y (value of y when $t = 0$)

average value

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

total distance traveled

$$\int_{t_1}^{t_2} |v(t)|\,dt$$

area bounded by two functions on $[y_1, y_2]$

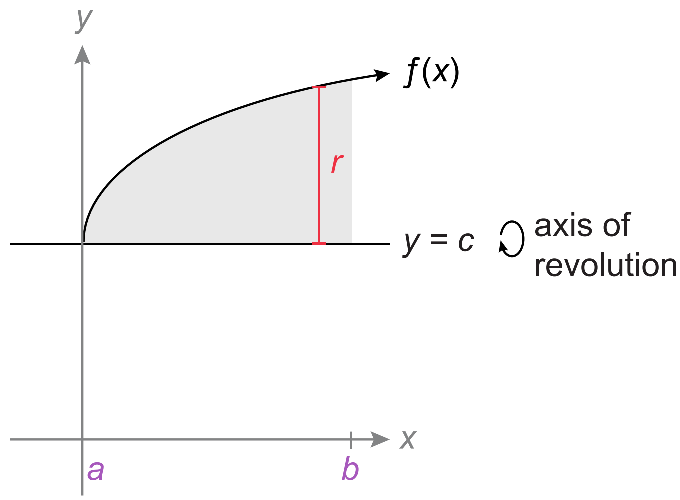
$$A = \int_{y_1}^{y_2} (\text{right} - \text{left})\,dy$$

area bounded by two functions on $[x_1, x_2]$

$$A = \int_{x_1}^{x_2} (\text{top} - \text{bottom})\,dx$$

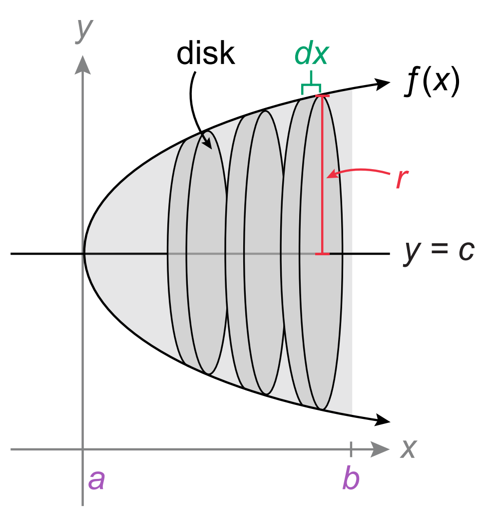
Use the **disk method** to determine the volume of a solid of revolution formed by rotating a region about a horizontal line $y = c$ (axis of revolution) over the interval $a < x < b$ when $y = c$ is a boundary of the region—there is no space between the region and $y = c$.

$$\pi \int_a^b r^2 \, dx$$



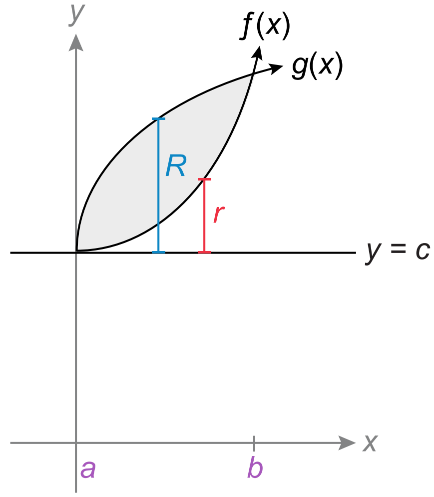
When a region is revolved about an axis of revolution, a perpendicular cross section of the solid is a disk where:

- r is the distance from the axis of revolution to the closest function $f(x)$
- dx is the thickness of the disk



Use the **washer method** to determine the volume of a solid of revolution formed by rotating a region bounded by $f(x)$ and $g(x)$ about a horizontal line $y = c$ (axis of revolution) over the interval $a < x < b$ when $y = c$ is not a boundary of the region—there is space between the region and $y = c$.

$$\pi\int_a^b ((R(x))^2 - (r(x))^2)dx$$



When a region is revolved about an axis of revolution, a perpendicular cross section of the resulting solid is a disk with a hole (washer) where:

- R is the distance from the axis of revolution to the farthest function $f(x)$
- r is the distance from the axis of revolution to the closest function $g(x)$
- dx is the thickness of the washer